Expected Returns and Liquidity Premium on the Paris Bourse: an Empirical Investigation

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Abstract

Liquidity has become an important factor in portfolio management. The existence of a liquidity premium according to which the return of low liquidity stocks is superior to the return of high liquidity stocks has been demonstrated. This article is an empirical study of the link between expected return, risk, and liquidity. A number of the main liquidity variables, as they emerge from theoretical breakthroughs of the microstructure literature as well as from portfolio management practices, and their measurement are presented for a sample consisting of the 120 main stocks quoted on the Paris Stock Exchange between July 1991 and August 1996. Liquidity premiums are estimated for portfolios with identical risk levels, from both a univariate and multivariate approach. Evidence is presented of a negative link between risk-adjusted expected returns and free float on the one hand, and of a positive link between risk-adjusted expected returns and relative spread on the other. The links between expected return, risk, and liquidity are significant even outside of the month of January, a notable divergence from results obtained on the NYSE. The liquidity premium estimated on a monthly basis is not constant. When the liquidity premium has a high absolute value, the observed rate of return is higher for the portfolio with the minimum free float during the year following the observation of the liquidity premium. This corroborates our measurements of liquidity and liquidity premiums and gives evidence of the usefulness of liquidity premiums in tactical asset allocation.

Résumé

La liquidité est un thème auquel les gestionnaires de portefeuilles accordent une grande importance compte tenu notamment des contraintes dans l'exécution des ordres. Les autorités boursières y sont également sensibles : en témoignent à Paris sur le système CAC la mise en place des contrats d'animation en 1992 et l'instauration d'un marché de blocs en 1994. Les recherches académiques sont en revanche peu nombreuses. Une déconnexion entre les littératures théorique et empirique est de plus sensible : les définitions théoriques se heurtant à des problèmes de mesure. L'objet de cette recherche est empirique. A partir de données horodatées de 1991 à 1996 sur le système CAC, différentes mesures de liquidité sont estimées. Une prime de liquidité mensuelle est évaluée à partir de taux de rentabilité anticipé. Un lien est mis en évidence entre le taux de rentabilité anticipé et la liquidité mesurée par la fourchette relative ou le flottant. Enfin, il est montré que ce résultat aurait pu être utilisé de manière profitable en allocation tactique de portefeuille : les mois où la prime de liquidité anticipée est forte sont suivis d'une progression exceptionnelle des cours des actions faiblement liquides. Ces derniers résultats, en l'absence de théorie reliant de manière satisfaisante la liquidité, le risque et la rentabilité exigée, corroborent les choix faits en matière de mesure de la liquidité et de la prime de liquidité.

1. INTRODUCTION

Consideration of the degree of liquidity of a market or a stock is becoming more and more widespread. Institutional investors are concentrating their orders on CAC 40 Index stocks, and research in some financial institutions concentrates exclusively on large stocks. The degree of accessibility and cost of information is one reason for this phenomenon. Another reason is liquidity and expected impact of an order on stock market prices; this constitutes the purpose of this paper.

In terms of the organization of markets and exchanges, liquidity was perceived early on as a desirable objective. This translated into the modification of quotation or exchange modalities for specific types of stocks. In Paris, various systems differed from Napoleon's agent market, from the first steps of block trading in 1972 to the launch of the block market in September 1994, passing through the liquidity contracts which began in 1992 or the transfer from continuous quotation to bi-daily fixing adopted in December 1991 for a few hundred less liquid stocks.

The developments in portfolio theory of the past thirty years are mostly concerned with risk and determining a risk premium. In comparison, the link between required return and stock liquidity has not been documented, with the exception of Treynor (1978). However, microstructure's research has made remarkable advances at both the theory level (with Kyle's ground-breaking essay (1985)) and the empirical level, with the development of ever more extensive databases. Many microstructure issues are concerned with the topic of liquidity.

There is nevertheless a disconnection between models and empirical measurements of liquidity. Empirical research has preferred conjunctural liquidity indicators, such as the relative spread (Amihud and Mendelson (1986)).

If a stock is very liquid, a high volume of transactions not motivated by information will not affect the stock price. According to Keynes, liquidity is "the ease with which a large trade can be executed without affecting prices significantly."

For Black (1971), the market of a stock is liquid if the following conditions are met: a spread, which is narrow, is permanently posted, large blocks can be traded at actual prices but during a time period proportional to the size of the block, and an immediate trade of a block results in a move proportional to the size of the block (in this last case, he is referring to opportunity cost).

The degree of liquidity of a stock can be analyzed as the result of the coexistence of orders driven by different motivations. The trader in a hurry (liquidity trader) is an uninformed agent who must realize a transaction before a given deadline (Harris, 1995, p.3). If the deadline is sufficiently far off and the spread is wide, he will have a tendency to place a limit order. A hurried trader doesn't necessarily behave irrationally: his behavior can equally be considered as the result of costly access to information. Handa and Schwartz (1996) make the distinction between patient investors, providers of liquidity, who mostly place limit orders, and hurried investors, asking for liquidity, who will place their orders at any price (market orders). The latter are at the root of excessive short-term volatility, and the strategies of the former are akin to strategies of volatility capturing. In other words, hurried investors give priority to the time at which the trade is carried out, while patient investors placing limit orders give priority to the price at which the trade is carried out. On a low-liquidity market, prices follow neither a random walk nor a martingale, rather, they tend to move away from their informational value and to come back to it with the liquidity bearers.

Illiquidity is clearly perceived as an inconvenience. The degree of aversion towards illiquidity is probably not the same from one investor to another, as it is linked to investment horizon, and therefore to management style. Chan and Lakonishok (1995) have opposed two management styles and have shown, on the basis of the transactions of 37 American institutional investors, that a group of impatient or aggressive investors (capital gains-oriented funds) registers a cost of 0.70% per round-trip trade on top of traditional commission costs, while the second group of more income-oriented funds registers a gain of 0.40%: the spread between the two groups comes out to 110 basis points.

The counterpart demanded for holding a low-liquidity stock by a very impatient investor with a very short-term investment horizon will be much higher than the counterpart demanded by a father looking to invest at an 18-year horizon upon the birth of his children. For Amihud and Mendelson (1986), the differences in horizon are at the root of the phenomenon of liquidity clienteles, with the investor with the shorter horizon overweighting his portfolio with liquid stocks and the investor with the longer horizon overweighting his portfolio with less liquid stocks. In equilibrium, a non-linear (concave) relation is then predicted between required return and stock liquidity. Of course, the analysis cannot ignore aversion towards risk and the advantages linked to diversification which interfere with the liquidity effect. The simultaneous modeling of these different parameters is quite a challenge, especially in a multiperiod framework. It is possible that the impact of transaction costs on required returns could be lightened, as shown by Constantinides (1986).

Liquidity may have some disadvantages because of the link between a dispersed shareholder structure and the liquidity of a stock. Dispersed shareholders may have more difficulty controlling management, and this could be the source of a cost justified by a stock's higher level of liquidity (Bhide, 1993). In this case, a return premium could be required for stocks with a very dispersed shareholder structure to compensate for agency costs induced.

But Holmström and Tirole (1993) point out the valuation advantages of greater liquidity and consequently, of a more dispersed shareholder structure. A dispersed structure is accompanied by an increase in the numbers and the activity of hurried traders (liquidity traders); therefore, the orders of informed investors are better hidden. The atomicity of the shareholder structure encourages the search for information, which in turn allows for better shareholder control of management.

The purpose of this paper is to propose measurements of liquidity and the liquidity premium. More specifically, it aims at answering the following questions:

- How may the liquidity of a stock be measured: many indicators are available, but how does one pick the least inadequate one? The difficulty here is that the "true" value of liquidity will remain an unknown: how does one pick between two or more estimators?
- Is an extra return required by shareholders for the holding of less liquid stocks? Is a discount applied to stocks perceived as having low liquidity? Does this premium evolve with time? Is it possible to determine a liquidity premium as it has been possible to demonstrate the existence of a risk premium?

The few available studies concern the American market and have used the relative spread as a measure of liquidity. But the relative spread includes an information asymmetry component for which it seems logical for the market to require an extra return.

This paper uses monthly expectational data for risk and return, in addition to continuous time-stamped market data to compute exogenous measures of liquidity from July 1991 to August 1996. It has six sections and is organized as follows: section 2 describes the databases that were used, as well as the liquidity variables that were selected; section 3 examines the relation between liquidity measurements and expected rates of return for portfolios of equivalent risk, and shows that the existence of a liquidity premium seems not to be limited to January; section 4 shows the liquidity premium from both a univariate and a multivariate approach; section 5 uses the liquidity premium for tactical asset allocation; and section 6 presents conclusions and perspectives.

2. DATA AND METHODOLOGY

The data used stem from two main sources of information:

- The expectational data of Associés en Finance as they are published monthly in its "Security Market Line" service. Depending on the month, endogenous expected risk, return, and liquidity figures are provided for 80 to 130 French stocks. The sample is not subject to an ex-post selection bias.
- The time-stamped data of the CAC system. The series is short but almost exhaustive, because although the CAC system was inaugurated in June 1986, its development was carried out progressively, with some underlying stocks of derivatives still quoted in continuous open outcry in 1990 (see Hamon and Jacquillat, 1992). Up until the end of 1994, stocks belong to the SBF 250 Index (data communicated by the SBF). From January 1995 to August 1996, the data are taken from the SBF's CD-ROM.

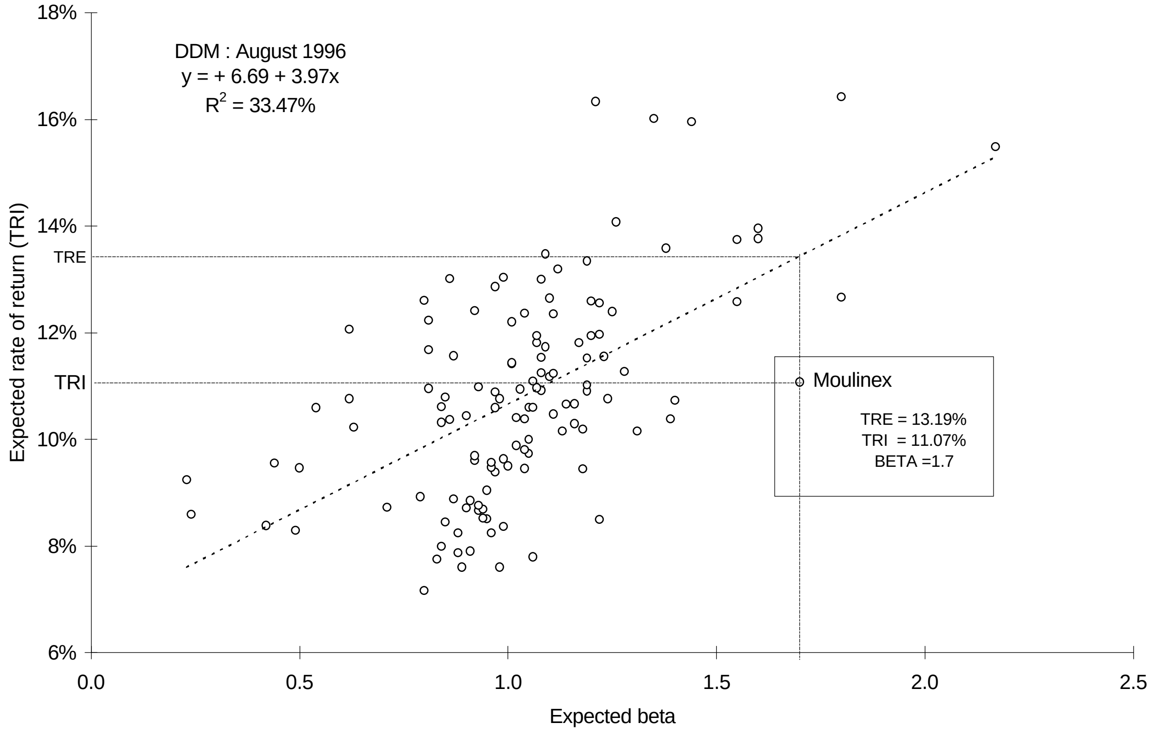
These two sources of data are described below.

2.1. Associés en Finance's Expectational Data

The Security Market Line projects each stock in a two-dimensional space with expected risk and return. The y-axis's required rate of return (TRI) is estimated by Associés en Finance by equalizing the observed stock price and the series of discounted future dividends estimated by financial analysts. These series are estimated from earnings per share and dividend forecasts for the next five years, and from a model beyond the fifth year. The x-axis's estimated risk is obtained on a monthly basis from a combination of four risk factors: the duration of the stock (calculated from the sequence of the stock's future dividends and of the market as a whole), the forecast risk, the financial risk, and the historical beta of the market sector to which the company belongs.

This representation results in a cloud of points. The slope of the adjustment line drawn through the cloud of points is the market risk premium (monthly estimated by ordinary least squares). Once the forecast risk is established, the equilibrium return in the CAPM sense is obtained for a given stock and month by a direct read; the Security Market Line gives the equilibrium rate of return (TRE).

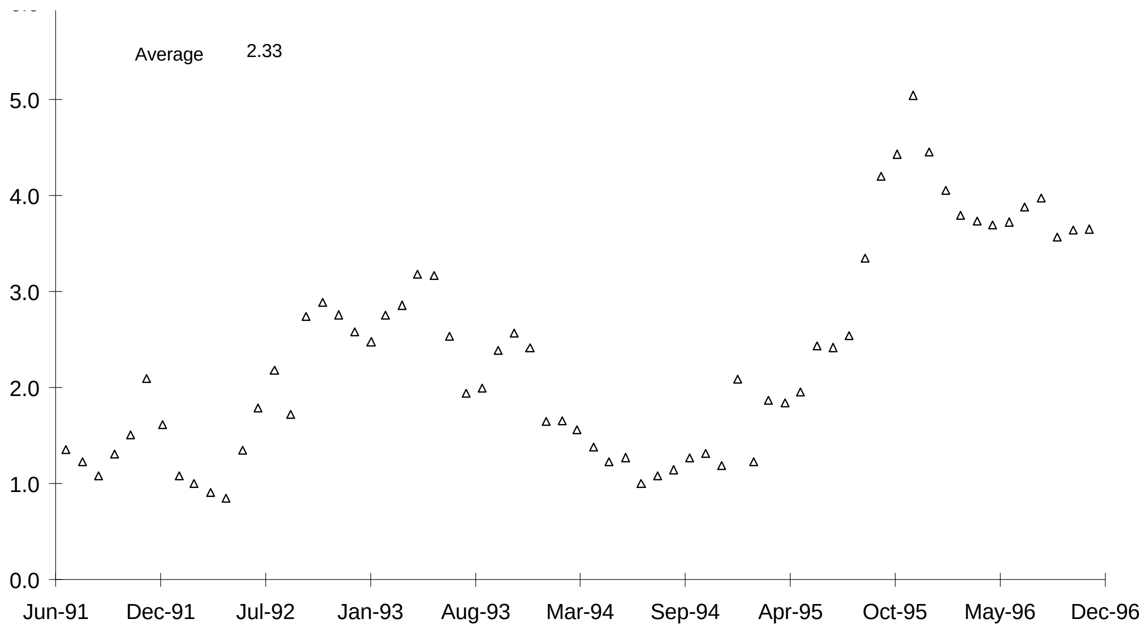
Figure 1
August 1996 Security Market Line



Note : The slope of the Security Market Line is estimated by the ordinary least square estimator. It measures the market risk premium.

As an indication, figure 1 represents the August 1996 Security Market Line. Each point represents a stock, such as Moulinex, the coordinates of which are indicated. The monthly estimate of the free float is also given by Associés en Finance. The risk premium is the slope of the line. FIGURE 2 shows the non-constancy of the estimated risk premium.

Figure 2
Monthly market risk premium



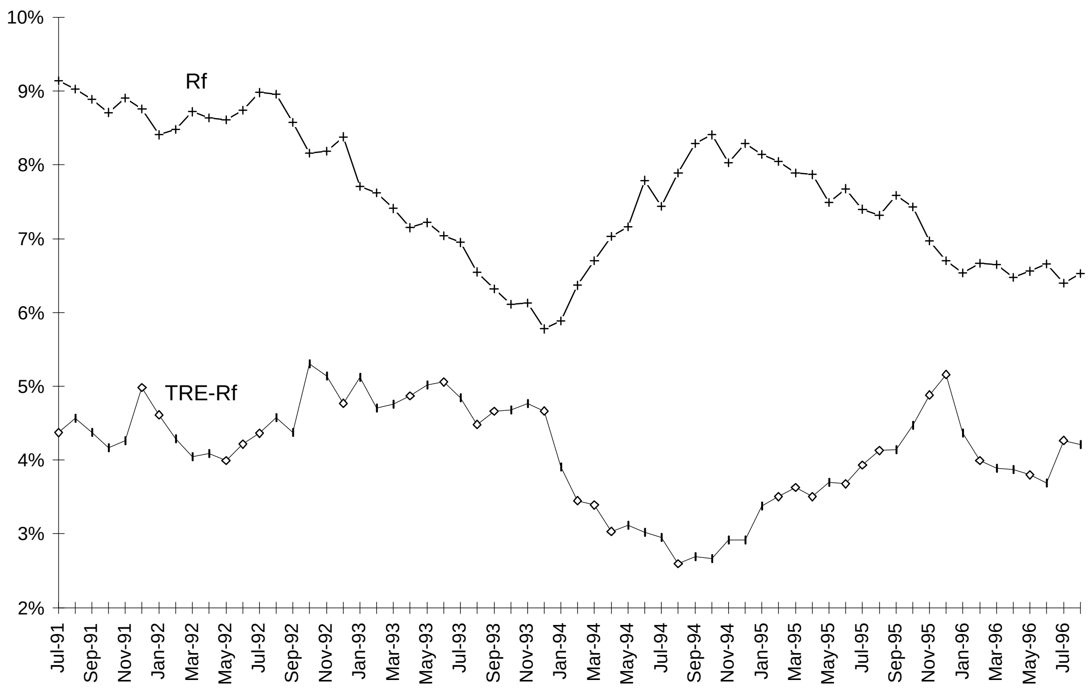
Note : The monthly market risk premium is estimated from July 1991 to August 1996.

The CAPM allows a split of the observed return in a given month in two parts : the first represents the normal return or the risk-adjusted equilibrium return (TRE), the other part is an excess return which reflects a misevaluation. The equilibrium rate of return (TRE) in the CAPM sense is given by the Security Market Line. If we know a given stock's ex ante risk, the SML directly gives the equilibrium rate of return (TRE). The distance for a given stock between the computed required rate of return (TRI) and the TRE is interpreted as a misevaluation.

FIGURE 3 represents the evolution of the risk free rate as estimated by the return of the French OAT (Obligations Assimilables du Trésor, the equivalent of US Treasury bonds) and the expected rate of return (in the CAPM sense) net of the risk free rate.

The free float represents the capitalization which is not frozen in portfolios as strategic investments: it is the part of total capitalization effectively available for ordinary transactions, estimated in millions of French francs. Although the CAPM doesn't deal with liquidity and liquidity premium as an explanatory factor of expected return, its empirical version as implemented by Associés en Finance computes a liquidity premium as the OLS regression coefficient of the equilibrium rate of return (TRE) of each stock on the log of its free float (expressed in millions of French francs).

Figure 3
Equilibrium rate of return evolution



2.2. *The Time-Stamped Data and Exogenous Liquidity Measures*

Estimating certain variables requires a thorough gathering of market data. Calculating a transaction volume means adding together the volumes associated with each basic transaction. Inevitably, 12 million transactions (times 4 basic pieces of information: time, price, amount exchanged, number of transactions) and 20 million best limits (times 7 basic pieces of information on the best high and low limits) had to be considered.

Each trading day was divided into seven time periods of one hour each. Transactions nearest the beginning of each period and the last transaction of the day were selected. Two spreads are determined for each of these transactions: the spread posted afterwards and the spread posted before or at the same time. For stocks in batch auction twice a day, this set of data is by definition limited to two sets of trading per session. The time-stamped database constructed in this way counts 947 944 transactions from July 1991 to December 1994 and 929 238 additional transactions from January 1995 to the end of July 1996.

There is a discrepancy between rigorous theoretical formulations, which are difficult to measure (for example, Kyle's lambda coefficient which involves the degree of dispersion of noise traders' orders), and the approaches frequently used by professional investors. The variables that can be used to quantify the liquidity phenomenon are largely intuitive and are all subject to some degree of criticism.

Appendix 1 describes in more detail the liquidity indicators used in this paper, which can be put in one of two categories: structural or conjunctural.

First among the structural indicators is the free float. Other variables are closely related: transaction volume, funds exchanged, and market capitalization. TABLE 1 specifies the correlation matrix between these variables.

Among the conjunctural indicators are the size of the transactions, the hourly turnover rate, the relative spread estimated as an average of hourly observations (the measurement is specified in the Appendix) [1]; the MEC, which is a variance ratio, the lambda, which measures the price's sensitivity to the amounts exchanged, and the flow, which is the ratio of volume to bid-ask size.

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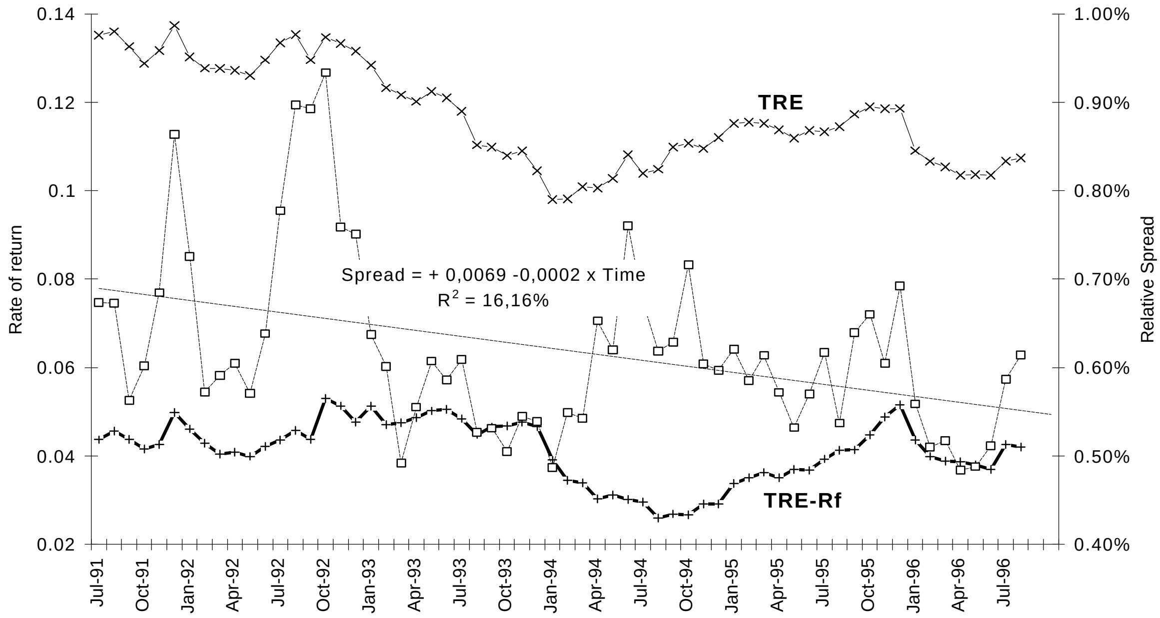
[1] Amihud and Mendelson (1986) estimate the relative spread from two observations carried out at the beginning and at the end of the year: it is doubtful that the spread in particular and liquidity in general can be considered as constant over a one year period.

FIGURE 4 shows the downward trend of the relative spread at the Paris Bourse since the beginning of the 1990s.

Figure 4
Evolution of the average relative spread and expected returns 1991 - 1996



Note: The intermediary line (right scale) tracks the evolution of the mean value of the stock spreads. The two other lines (left scale) show the evolution of the average equilibrium return rate in the CAPM sense (TRE) and the same rate minus the risk-free rate (TRE-Rf).

The turnover rate relates the funds exchanged at a given time to market capitalization. An aggregate is determined on both a daily and a stock-by-stock basis (arithmetic mean). The MEC relates the daily variance (from opening to close) to seven times the hourly variance. According to the random walk hypothesis, variance is proportional to the elapsed time and the ratio is equal to one. With the presence of liquidity traders, prices will move away from their informational value. This move will only be temporary because of the intervention of patient investors placing limit orders. The intervention of liquidity traders should therefore be at the root of excess short-term volatility and make the variance ratio fall below one.

The lambda is estimated under the hypothesis of a linear relation between the amounts exchanged and the variation of prices. A very liquid stock is characterized by a lambda equal to zero, and its price and informational value are the same at each instant. An illiquid stock (positive lambda) will go down if a transaction is initiated by a seller and will go up if a transaction is initiated by a buyer. The lambda is estimated from the coordinates of easily observable points given by the best limits of the order book.

Table 1
Correlation matrix between monthly estimates (July 1991 to August 1996)

Variable	abbr	n°	TRE	Rf	3	4	5	6	7	ftrel	9	10	11	12	13	14	15	Mec	17	freqt	19	débit	21	22
Equilibrium rate of return	TRE	1	1,00																					
Risk Free Rate (OAT)	Rf	2	0,79	1,00																				
Excess Return	TRE-Rf	3	0,57	-0,06	1,00																			
Zero-beta portfolio return	0-Beta	4	0,83	0,91	0,14	1,00																		
Required rate of return in exces of zero-beta portfolio return	TRE-0beta	5	-0,31	-0,68	0,40	-0,79	1,00																	
Required Rate of Return	TRI1	6	1,00	0,78	0,57	0,83	-0,31	1,00																
Expected Risk	Risque	7	-0,01	0,08	-0,11	0,25	-0,44	-0,02	1,00															
Relative Spread	Ftrel	8	0,61	0,60	0,19	0,55	-0,27	0,61	0,11	1,00														
Free Float	FlotM	9	-0,91	-0,84	-0,36	-0,88	0,48	-0,91	-0,12	-0,61	1,00													
Volume	Qtittran	10	-0,76	-0,57	-0,48	-0,73	0,41	-0,77	-0,17	-0,39	0,71	1,00												
Market Capitalization	CapitM	11	-0,78	-0,84	-0,15	-0,74	0,39	-0,78	0,08	-0,62	0,88	0,46	1,00											
Transactions	Capitaux	12	-0,28	-0,18	-0,20	-0,17	-0,02	-0,28	0,18	-0,09	0,21	0,16	0,20	1,00										
Price	Coursouv	13	-0,19	0,00	-0,31	0,24	-0,61	-0,19	0,45	-0,30	0,07	-0,24	0,32	0,14	1,00									
Transaction size	tailoa	14	-0,54	-0,38	-0,36	-0,63	0,47	-0,54	-0,22	-0,29	0,52	0,79	0,22	0,01	-0,50	1,00								
Lambda	lambda	15	-0,07	0,28	-0,48	0,05	-0,17	-0,06	-0,39	0,40	0,00	0,20	-0,28	-0,04	-0,19	0,24	1,00							
Market Efficiency Coefficient	MEC	16	-0,09	-0,16	0,07	-0,03	-0,04	-0,10	0,26	-0,28	0,03	0,15	0,14	0,04	0,14	0,00	-0,40	1,00						
Turnover	Tauxrot	17	-0,64	-0,73	-0,07	-0,75	0,57	-0,64	-0,27	-0,56	0,71	0,75	0,62	0,14	-0,11	0,55	-0,08	0,19	1,00					
Transaction Frequency	freqt	18	-0,81	-0,78	-0,28	-0,76	0,40	-0,82	-0,03	-0,44	0,80	0,74	0,74	0,31	0,13	0,33	-0,01	0,15	0,77	1,00				
Hourly return rate standard deviation	taux1	19	-0,07	0,12	-0,26	0,00	-0,07	-0,07	0,07	0,13	0,00	0,11	-0,03	-0,02	0,00	0,09	0,23	0,00	-0,07	0,02	1,00			
Flow	débit	20	-0,74	-0,78	-0,17	-0,87	0,66	-0,74	-0,17	-0,38	0,82	0,81	0,68	0,16	-0,24	0,59	0,02	0,15	0,84	0,85	0,08	1,00		
Taux de rentabilité quotidien	rentab	21	-0,02	-0,21	0,23	-0,08	0,12	-0,05	-0,04	-0,24	0,13	0,05	0,30	-0,11	-0,01	-0,06	-0,34	0,44	0,18	0,16	-0,06	0,16	1,00	
Daily rate standard deviation	srentab	22	0,29	0,18	0,24	0,18	0,01	0,28	0,06	0,68	-0,26	0,00	-0,24	-0,05	-0,30	-0,13	0,19	0,16	0,01	0,05	0,07	0,17	0,13	1,00

Note : Correlations are established from monthly arithmetic means on a stock-by-stock basis. Variables can either be measured as a punctual observation (TRE, capitalization), a daily average of punctual observations (price), or a daily average of hourly observations (spread, lambda, flow...). Appendix 1 describes each of these variables precisely.

The flow is estimated for a stock for a given hour by calculating the ratio of the number of stocks exchanged in the hour to the amounts in the order book at the best limits at the end of the period. The average flow of a stock is obtained by calculating the arithmetic mean of the hourly estimates.

3. RISK, RETURN AND SPREAD

Amihud and Mendelson (1986) assimilate liquidity and relative spread and have established a positive but decreasing relation between return and spread. The relation is positive which is compatible with a hypothesis of aversion towards illiquidity ; but the slope decrease, which is compatible with a hypothesis of a liquidity clientele. The test is carried out from 1961 to 1980 on ex-post data, with the spread associated with one year for one stock being the mean figure derived from the two figures of the beginning and the end of the preceding year.

Eleswarapu and Reinganum (1993) duplicate Amihud and Mendelson's results while widening the period under study and analyzing the results according to specific months. Data are available from 1951 to 1980 for between 654 and 929 stocks depending on the year. The test period is 1961-1980. For each estimation period, the authors create 49 equally weighted portfolios and distribute the shares according to their beta and their spread. From year 1 to 5, beta are estimated (with equally weighted index and excess return); from year 6 to 10: 7 portfolios are constructed following the value of the beta, with 7 portfolios with different spreads constituted year 10, inside each risk portfolio.[2] Exactly in 1961, the first year of tests comes in at the eleventh year and 7x7=49 portfolios are available. The tests are carried out on one year periods and they match the portfolio betas, the returns, and the spreads estimated the preceding year. With the test period stretching from 1961 to 1980, 980 portfolios with an approximately identical size are defined. The results show an increasing relation between return and beta and an increasing relation between return and spread. But the relation is only significant in January and is not

[2] The period starts in 1951, betas are estimated and portfolio constituted from 1951 to 1955, the risk of the portfolios are re-estimated from 1956 to 1960, and the spread portfolios are formed in 1960.

supported in other months. Methodological variations (regressions) yield qualitatively identical results.

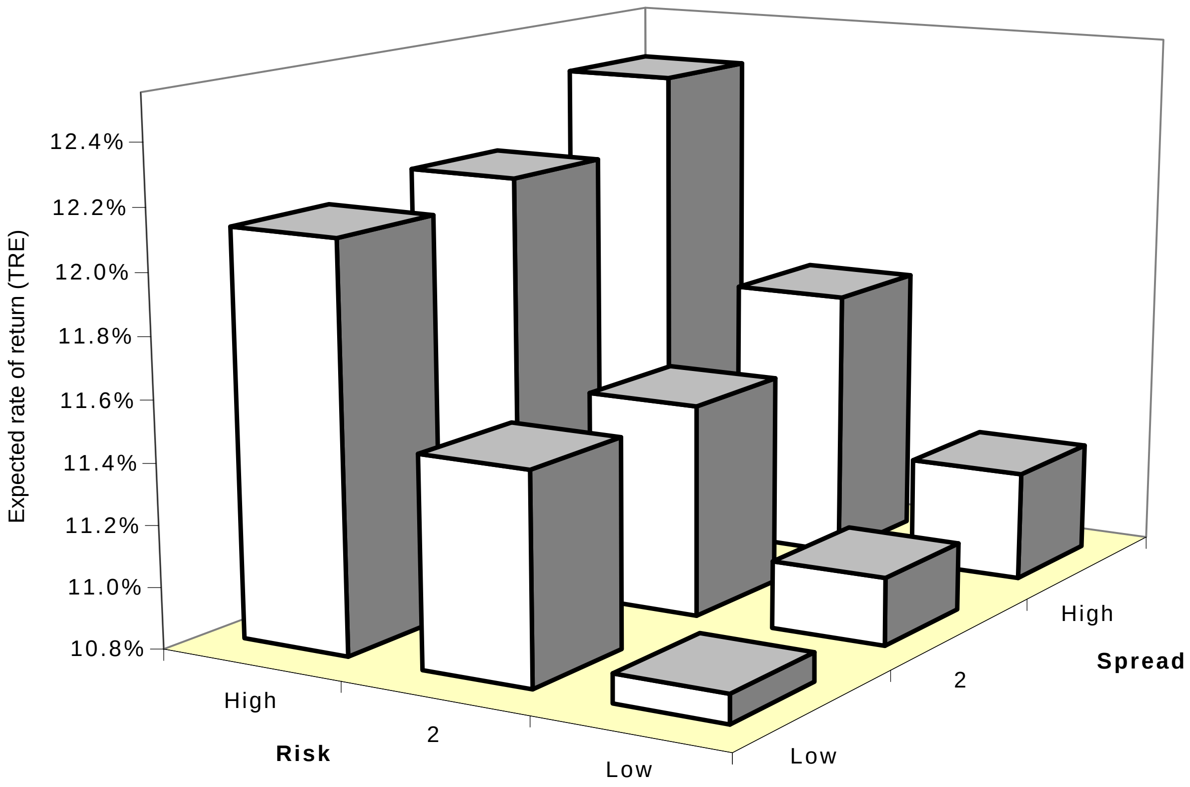
The relatively short length of the available period in this study requires a different methodology from the one used in the above papers to test the link between rate of return, risk, and liquidity. The earliest available historic spread data for the Paris Bourse is March 1990. One alternative is to use expectational risk and return data.

Each month from early July 1991 to the end of August 1996, available stocks are split into three classes according to the value of their spread as it is observed that month. The three classes are formed so as to contain approximately the same number of stocks. Within each spread class, the stocks are split monthly into three risk classes. Therefore, each month, 3x3=9 classes are constituted.

3.1. Results from July 1991 to July 1996 with the relative Spread

The equilibrium rate of return is positively linked to risk (be it forecast -figure 5- or estimated by volatility -figure 6) and positively linked to the spread.

Figure 5
Expected return, expected risk, and quoted spread



The equilibrium rate of return is negatively linked to the free float (figure 7) and the market capitalization (figure 8). The Student tests are the strongest for the link between return, risk, and free float (table 3). [3]

Table 2
Expected return, expected risk, and quoted spread

		Class of expected risk			All	T-test
		Low	2	High	stocks	(high-low)
Class of relative spread	Low	10.84% 758 (1.30%)	11.43% 696 (1.10%)	12.07% 686 (1.14%)	11.43% 2140 (1.29%)	19.14
	2	10.97% 696 (1.24%)	11.45% 696 (1.10%)	12.13% 748 (1.16%)	11.53% 2140 (1.26%)	18.23
	High	11.11% 709 (1.23%)	11.65% 709 (1.08%)	12.35% 778 (1.25%)	11.72% 2196 (1.30%)	19.24
	All stocks	10.97% 2163 (1.26%)	11.51% 2101 (1.10%)	12.19% 2212 (1.19%)	11.56% 6476 (1.29%)	32.73
T-test	(high-low)	4.05	3.65	4.47	7.49	

Note: Each month from July 1991 to August 1996, the stocks are divided among three classes following the value of the posted spreads (mean of the hourly observations) and within each spread class following the value of the estimated risk. The table gives the mean of the estimated rates of return (TRE), the number of observations, and the standard deviation, the latter in parentheses.

[3] A partitioning into 16 classes (4 according to the value of the spreads and 4 according to the value of the forecast risk) yields qualitatively identical results with approximately identical Student tests values. A partitioning into sub-periods also confirms the results, particularly the first test which was carried out on the data for the period from January 1995 to February 1996. The small number of observations weakens the results but the Student tests on return differences between classes of extreme illiquidity are significant in 3 out of 4 cases. These results are not detailed here.

Figure 6
Expected return, ex-post volatility and quoted relative spread

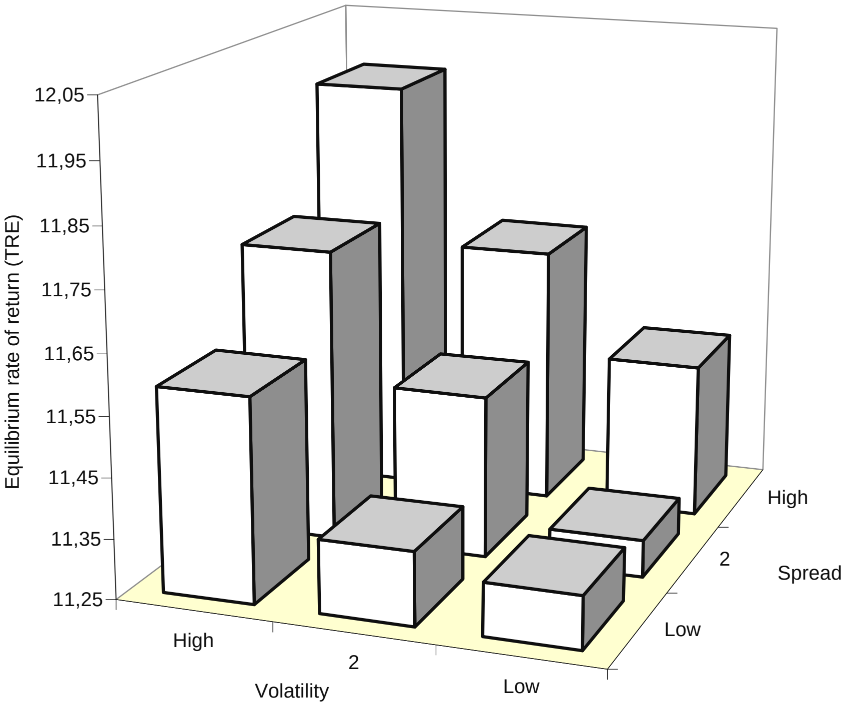


Figure 7
Expected return, risk, and free float

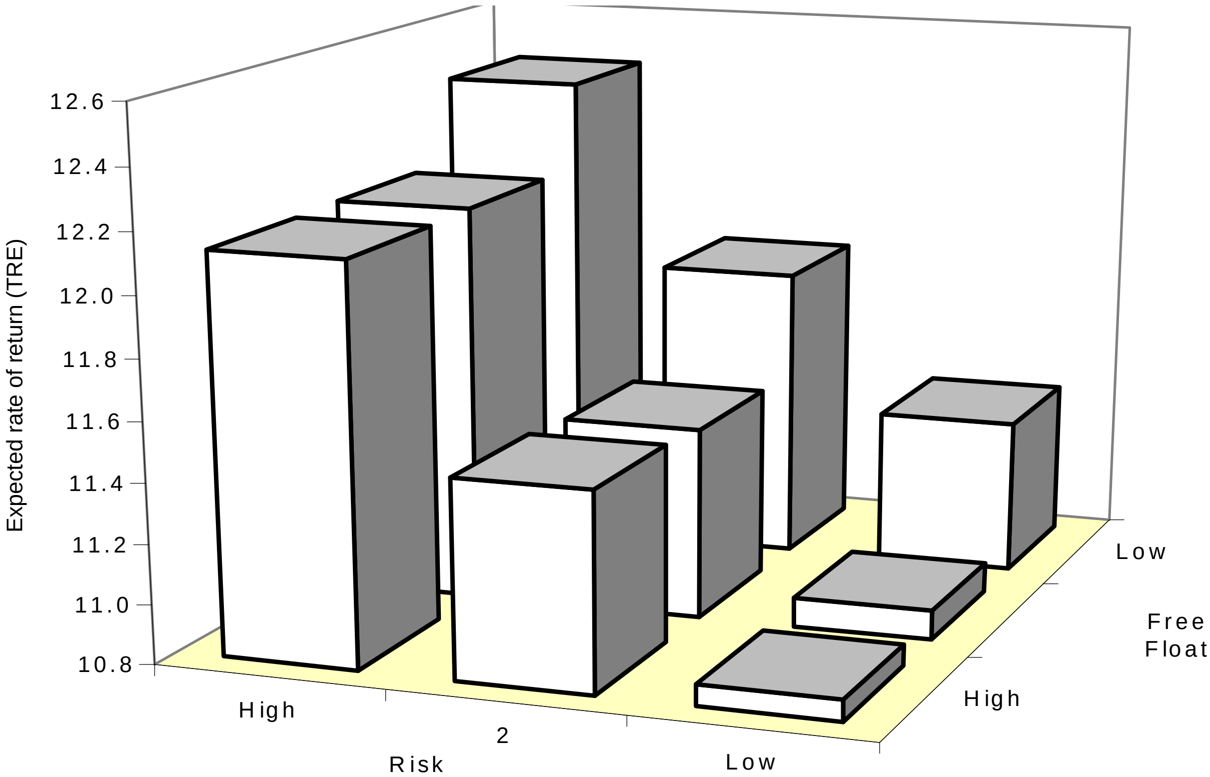


Table 3
Expected return, risk, and free float

Free float Class	Risk Class Low	2	High	All stocks	T test high-low
	11.27	11.74	12.37	11.78	17.54
	758	696	686	2 140	
Low	(1.16)	(1.08)	(1.21)	(1.24)	
	10.85	11.39	12.08	11.46	18.60
2	696	696	748	2 140	
	(1.30)	(1.11)	(1.22)	(1.31)	
High	10.82	11.41	12.08	11.46	19.68
	709	709	778	2 196	
	(1.29)	(1.09)	(1.16)	(1.29)	
All stocks	10.99	11.52	12.17	11.56	31.65
	2 163	2 101	2 212	6 476	
	(1.26)	(1.10)	(1.20)	(1.29)	
T test (low-high)	7.01	5.64	4.69	8.29	

Figure 8
Expected return, risk, and capitalization

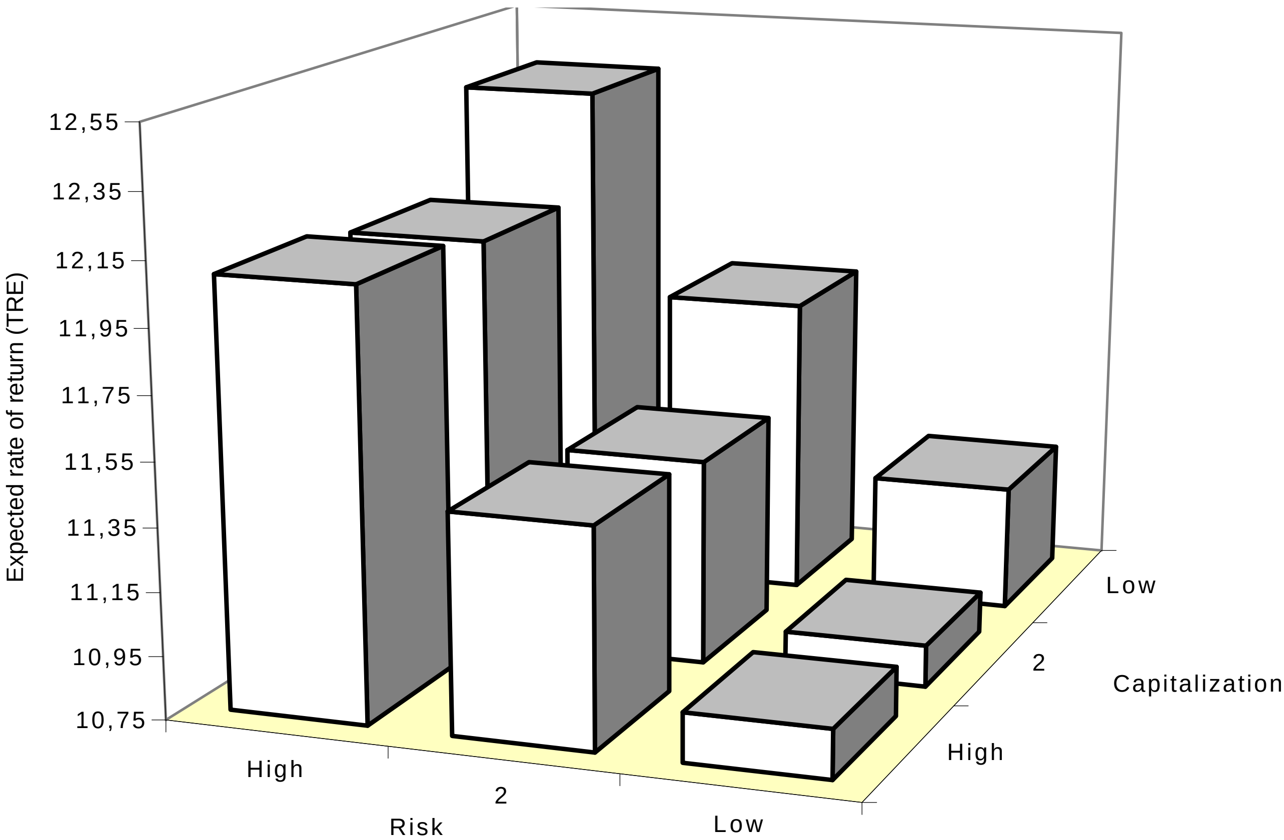


Table 4
Expected return, risk, and capitalization

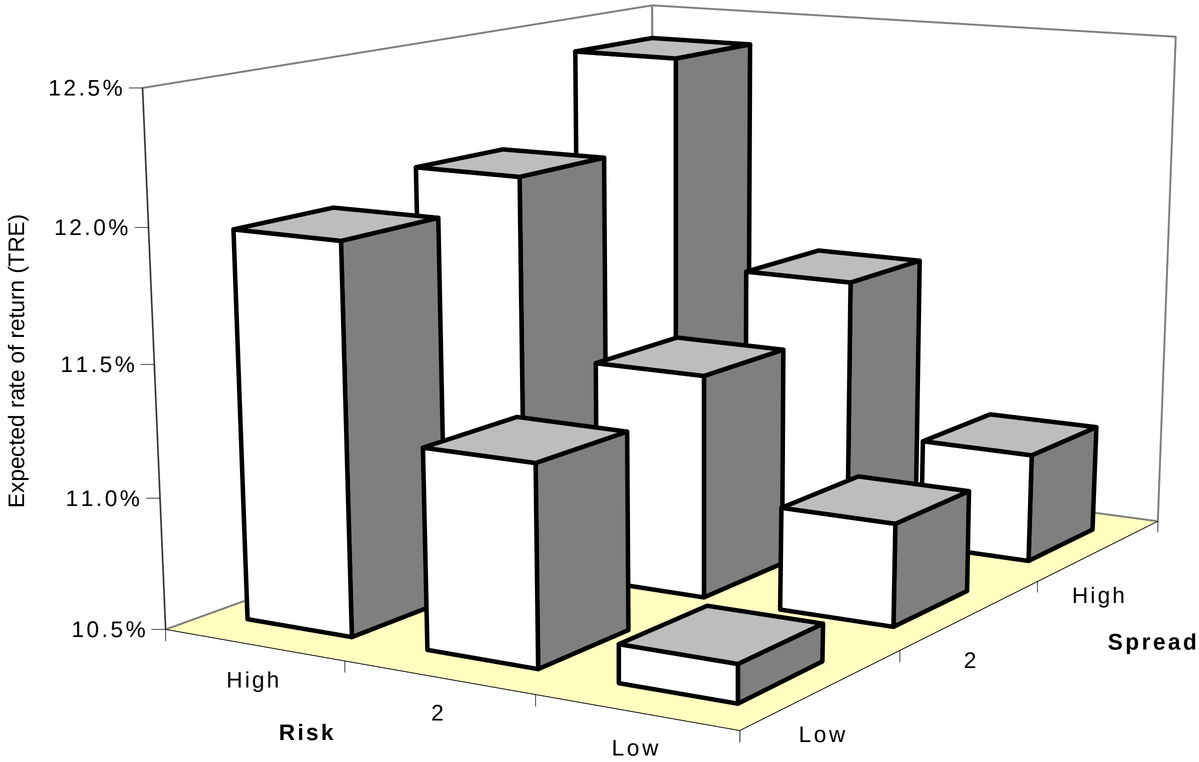
Capitalization Class	Risk class Low	2	High	all stocks	T test (high-low)
	10.90	11.44	12.09	11.46	18.95
High	758	696	686	2 140	
	(1.25)	(1.09)	(1.13)	(1.26)	
2	10.88	11.40	12.05	11.46	17.09
	696	696	748	2 140	
	(1.33)	(1.12)	(1.26)	(1.33)	
Low	11.15	11.71	12.38	11.76	19.93
	709	709	778	2 196	
	(1.18)	(1.06)	(1.19)	(1.25)	
All stocks	10.98	11.52	12.18	11.56	32.18
	2 163	2 101	2 212	6 476	
	(1.26)	(1.10)	(1.20)	(1.29)	
T test (Low-High)	3.83	4.73	4.70	8.02	

3.2. January and the Rest of the Year

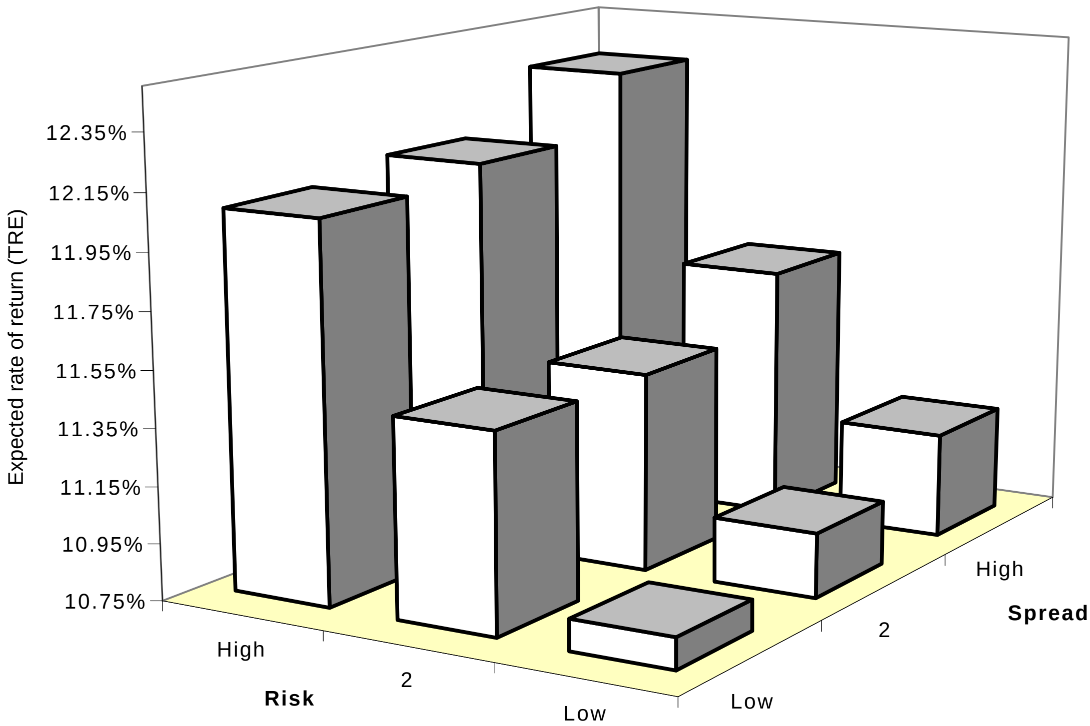
Eleswarapu and Reinganum (1993) observed on the NYSE, very different results following the month of the year from 1961 to 1990. All months combined, the authors found positive relationship between the ex-post rate of return and the risk or the degree of illiquidity (measured by the relative spread). But a closer look at the results shows that this link can only be confirmed in the month of January!

The results presented in figure 9 and table 5 are a breakdown according to the month of the year and show that contrary to the US case, the relation between rate of return, risk, and spread is qualitatively identical in all months of the year.

Figure 9
Expected return, risk, and spread (July 1991 - August 1996)
Panel A : in January



Panel B : February to December



Note : the two figures are drawn with the data of table 4.

Table 5
Expected returns, risk, and quoted spread

Panel A : January, 1992 to 1996

Spread class	Risk Class Low	2	High	All stocks	T test (high-low)
Low	10,64%	11,25%	11,97%	11,27%	4,91
	53	48	49	150	
	(1,46%)	(1,27%)	(1,27%)	(1,44%)	
2	10,90%	11,36%	12,05%	11,46%	4,44
	48	48	54	150	
	(1,31%)	(1,27%)	(1,32%)	(1,38%)	
High	10,93%	11,55%	12,39%	11,64%	5,11
	50	50	53	153	
	(1,41%)	(1,25%)	(1,50%)	(1,51%)	
All stocks	10,82%	11,39%	12,14%	11,46%	8,39
	151	146	156	453	
	(1,40%)	(1,26%)	(1,37%)	(1,45%)	
T test (high-low)	1,02	1,15	1,55	2,17	

Panel B : February to December, 1991 to 1996

Spread class	Risk Class low	2	high	All stocks	T test (high-low)
Low	10,86%	11,45%	12,08%	11,44%	18,51
	705	648	637	1 990	
	(1,28%)	(1,08%)	(1,13%)	(1,27%)	
2	10,98%	11,45%	12,13%	11,54%	17,70
	648	648	694	1 990	
	(1,24%)	(1,09%)	(1,15%)	(1,25%)	
High	11,12%	11,65%	12,35%	11,73%	18,55
	659	659	725	2 043	
	(1,21%)	(1,07%)	(1,24%)	(1,28%)	
All stocks	10,98%	11,52%	12,19%	11,57%	31,67
	2 012	1 955	2 056	6 023	
	(1,25%)	(1,08%)	(1,18%)	(1,27%)	
T test (high-low)	3,93	3,47	4,19	7,16	

Note : The table indicates, for each class, the mean TRE, the number of observations, and the standard deviation (in parentheses). The far-right column presents the results of the Student tests on the mean difference of TRE between the two extremes classes of risk. The bottom line indicates the results of the Student tests on the mean difference of TRE between the two extremes classes of relative spread.

4. LIQUIDITY PREMIUM:

A UNIVARIATE AND A MULTIVARIATE APPROACH

In this section, we measure the liquidity premium and investigate various explanatory variables with two econometric approaches: univariate and multivariate. If it exists, and there is every reason to believe so, the liquidity premium is considered as the differential return required between more liquid and less liquid stocks. According to the literature, the liquidity premium should be negative. The liquidity measures we investigate are the relative spread, the free float and the market capitalization.

In the univariate approach, the expected return (TRI) is regressed once at a time with each of these variables. We expect the sign of the regression coefficient to be positive for the relative spread and to be negative for both other liquidity variables.

In a second stage we use the multivariate approach, and regress the expected return (TRI) with each of the liquidity measures one at a time, and each time with the risk measure, in order to check whether risk is not another proxy for liquidity.

We start with the equilibrium model of capital assets (CAPM):

$$E[R_i] = R_f + b_i \times \left(E[R_M] - R_f\right)$$

$$\text{where } \mathrm{E}[\mathrm{R_i}] = \mathrm{a_0} + \beta_\mathrm{i} \times \mathrm{a_1}$$

a_0 and a_1 are estimated monthly in least ordinary squares; the results are in table 6. a_1 is the market risk premium.

$$\mathrm{E}[\mathrm{R_i}] = \mathrm{a_0} + \mathrm{Ftrel_i} \times \mathrm{a_2}$$

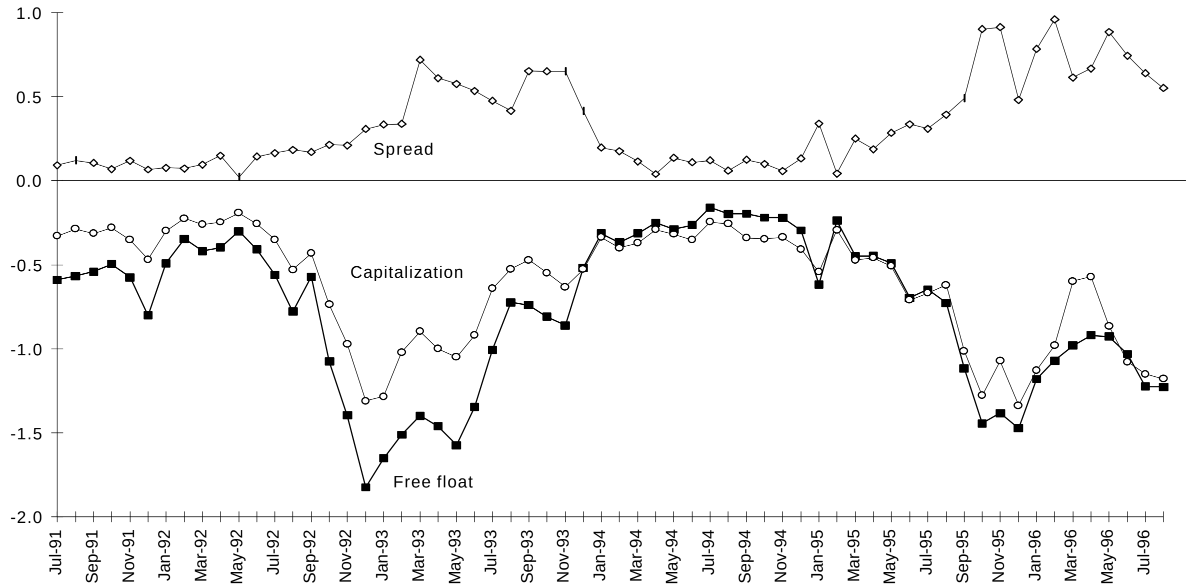
where Ftrel represents the relative spread and a_2 estimates the liquidity premium.

Table 6
Univariate risk and liquidity premium

Year	Nb obs	Risk premium	Liquidity premium estimated with: relative spread				free float			
			Slope*10^{-2}	T(slope)	R^2	F	Slope	T(slope)	R^2	F
1991	510	1.33	0.1824	4.21	3.4%	17.7	-0.66	-3.23	2.0%	10.4
1992	1,038	1.46	0.2827	8.34	6.3%	69.6	-0.75	-4.95	2.3%	24.5
1993	1,097	2.74	0.6995	7.07	4.4%	50.0	-1.22	-4.82	2.1%	23.3
1994	1,380	1.58	0.2630	5.14	1.9%	26.4	-0.41	-3.00	0.6%	9.0
1995	1,457	2.65	0.4907	8.00	4.2%	64.0	-0.83	-4.90	1.6%	24.0
1996	994	3.58	0.7647	7.30	5.1%	53.3	-1.09	-5.09	2.5%	25.9

Note: Annual estimates only are reported in this table. They are derived each year from all available observations (stock*month).

Figure 10
The liquidity premium with a univariate approach: a synthesis



Note : This graph represents the evolution of the monthly estimates of liquidity premiums. The figures are produced using three approaches to liquidity: free float, capitalization, and relative spread.

Using the hypothesis of linearity in a multivariate approach, a joint estimate of the liquidity premium and the risk premium is made with an OLS regression from the following model:

$$E\left[R_i\right] = a_0 + b_i \times a_1 + Ftrel_i \times a_2$$

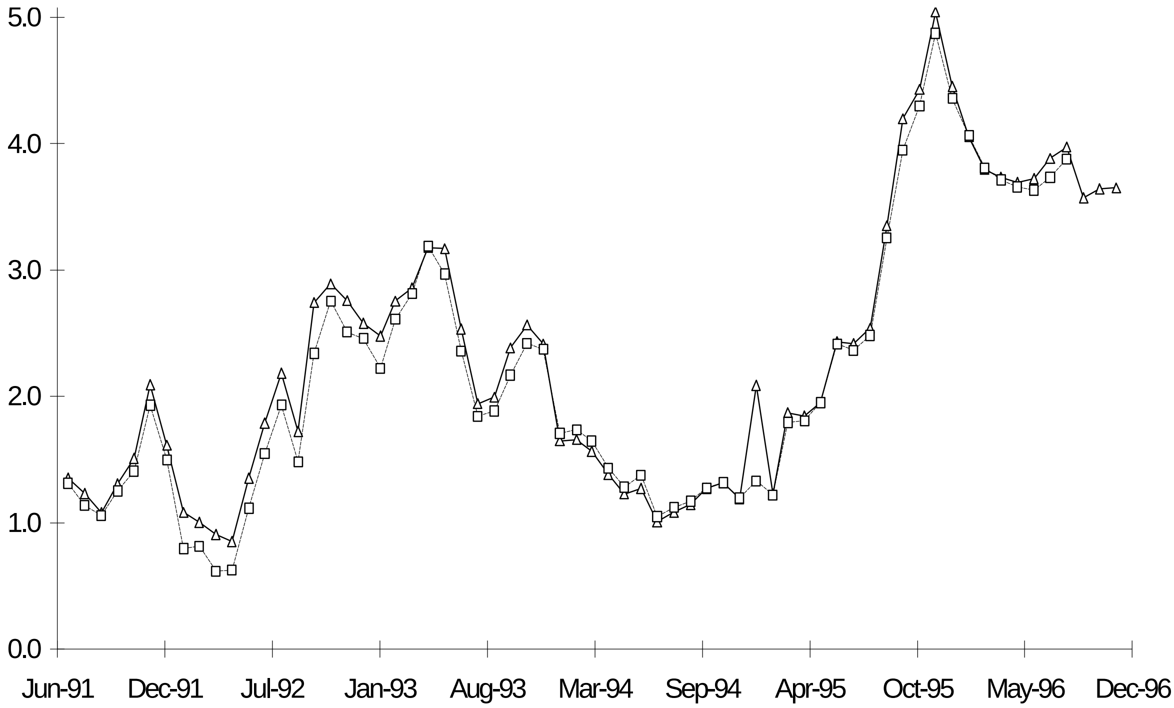
Ftrel represents the spread of stock i for a given month, E[Ri] is the rate of return and b is the anticipated risk. The risk premium is represented by a_1 . The liquidity premium is represented by a_2 . The estimate is made every month from July 1991 to August 1996; the results appear in table 7.

Table 7
Risk premium and liquidity premium

	Liquidity premium					Risk premium		
Year	Slope	σ	T test	R^2	F	Slope	σ	T test
1991	7.4	15	0,50	5,7%	15,3	1.42	0.26	5,41
1992	40.3	10	3,97	10,7%	61,7	1.64	0.17	9,55
1993	71.4	13	5,54	24,4%	176,3	2.38	0.14	16,47
1994	-10.3	10	-1,05	7,9%	59,4	1.30	0.12	10,90
1995	59.2	11	5,62	24,6%	237,5	2.61	0.13	19,78
1996	34.2	16	2,20	36,4%	283,7	3.83	0.17	22,74

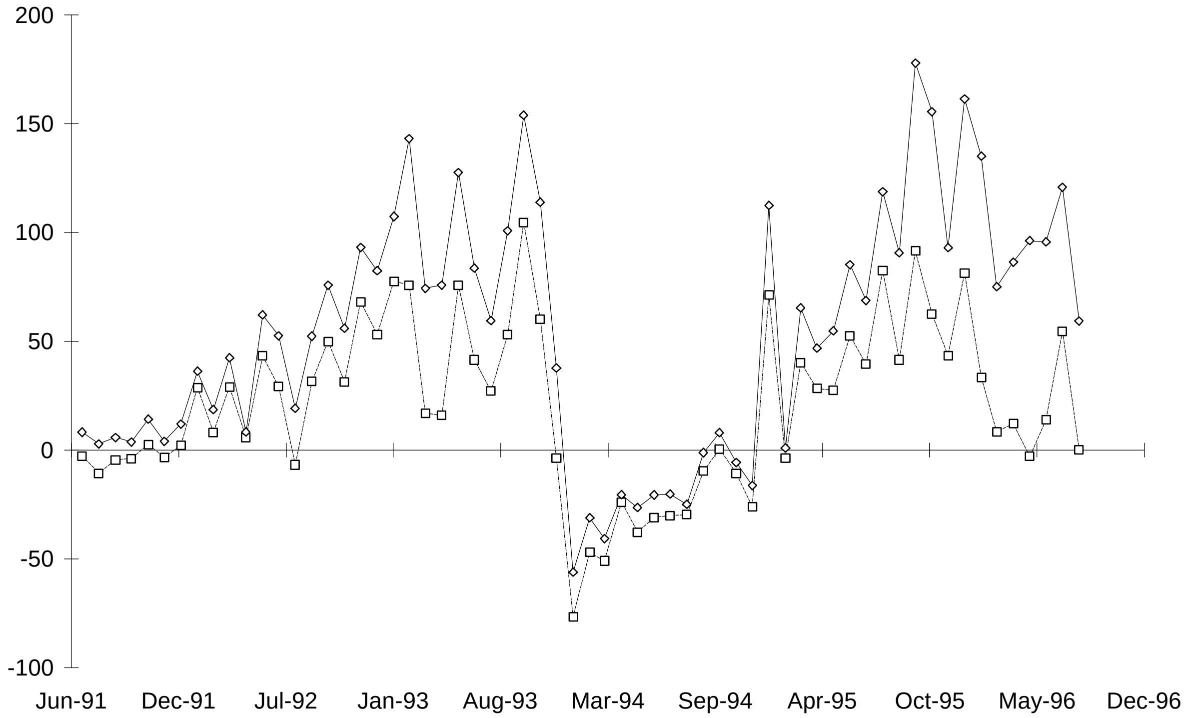
Note : Annual estimates only are reported in this table. They are derived each year from all available observations (stock*month).

Figure 11
Risk premium : comparison between univariate and multivariate estimates



Note : The dotted line represents stocks with a univariate evaluation; the solid line represents the multivariate evaluation (liquidity estimated by the spread). The correlation coefficient between the two series is 99.14%. This graph shows the evolution of monthly risk premium estimates.

Figure 12
Liquidity premium: a comparison



Note : The dotted line represents stocks with a multivariate evaluation; the solid line represents the univariate evaluation. The correlation coefficient between the two series is 0.92.

5. LIQUIDITY PREMIUM AND TACTICAL ASSET ALLOCATION

This section tests whether the liquidity premium has any use in terms of tactical asset allocation. When the liquidity premium is unusually high or low, portfolios are formed according to the liquidity of individual stocks, and the subsequent risk-adjusted returns are computed.

Table 8
Free float and ex-post return (all months)

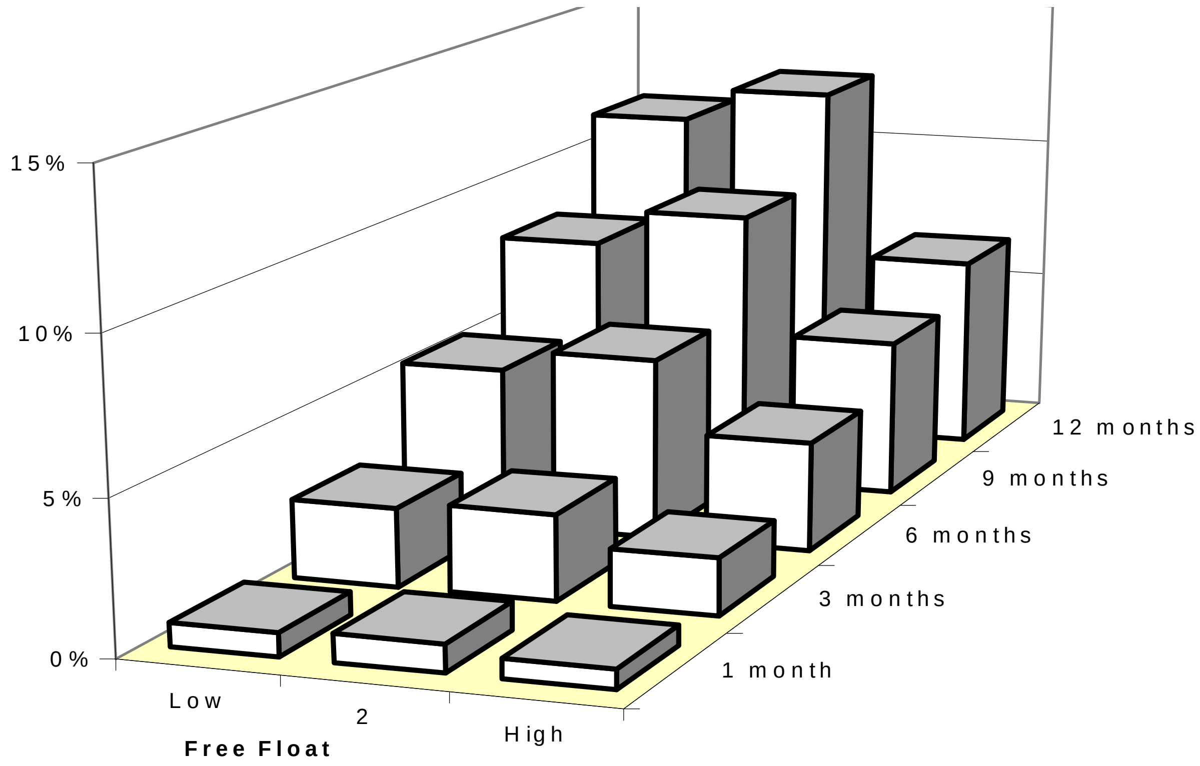
	Free Float			All	T test
	High	2	Low	Stocks	(low-high)
1 month	0.64%	0.91%	0.76%	0.77%	0.51
3 months	1.88%	2.84%	2.60%	2.43%	1.71
6 months	3.64%	6.04%	5.33%	5.00%	2.80
9 months	5.27%	9.33%	8.13%	7.57%	3.68
12 months	6.53%	12.38%	11.22%	10.02%	4.79

Each month, from July 1991 to August 1996, 3 portfolios are formed following the stocks' free float. The rates of return of these 3 portfolios are computed for the following months for a period of one year after the formation date (the last available month is December 1996). After

6 months, there is a significant difference between the returns of the low free float and high free float portfolios. One year after the formation date, the average rate of return of the low free float portfolio is 11.22% against 6.53% for the high free float portfolio. This first result is simply due to the small size effect.[4]

From July 1991 to August 1996, the average monthly liquidity premium estimated with the free float is -0.43 (table 9). In order to see whether the liquidity premium could be used as a tactical asset allocation indicator, we formed three sets of months according to the value of the liquidity premium estimated with the free float. The first set included 24 months where the liquidity premium was lower than -0.59 (the average value minus half of the standard deviation), the second set includes 19 months with a liquidity premium above -0.27 (the average plus half of the standard deviation). The third set included all the other months (intermediate liquidity premiums).

Figure 13
Ex-post return for portfolios of different free float



[4] The small size effect is also observed using capitalization or relative spread. If portfolios are formed according to the relative spread of the stocks, the rate of return of the high relative spread portfolio is 8.87% one year after the formation month, against only 6.32% for the low relative spread portfolio: the difference is significant, with a T-test of 2.47.

Note : Each month, 3 portfolios are formed following the stocks' free float. The 3 portfolios' rates of return are computed for the following months for a period of one year after the formation date.

Table 9
Monthly liquidity premium (July 1991 to August 1996)

Jul-91	-0.24	Sep-92	-0.73	Nov-93	-0.62	Jan-95	-0.97	Mar-96	-0.33
Aug-91	-0.29	Oct-92	-1.03	Dec-93	-0.36	Feb-95	-0.12	Apr-96	-0.41
Sep-91	-0.18	Nov-92	-0.68	Jan-94	-0.01	Mar-95	-0.43	May-96	-0.46
Oct-91	-0.25	Dec-92	-0.78	Feb-94	0.05	Apr-95	-0.27	Jun-96	-0.62
Nov-91	-0.36	Jan-93	-0.59	Mar-94	0.15	May-95	-0.18	Jul-96	-0.74
Dec-91	-0.52	Feb-93	-0.76	Apr-94	0.08	Jun-95	-0.25	Aug-96	-0.60
Jan-92	-0.39	Mar-93	-0.65	May-94	0.10	Jul-95	-0.37	Average	-0.43
Feb-92	-0.67	Apr-93	-0.51	Jun-94	0.31	Aug-95	-0.39	σ	0.32
Mar-92	-0.49	May-93	-0.50	Jul-94	0.15	Sep-95	-0.56	Nb obs	61
Apr-92	-0.66	Jun-93	-0.80	Aug-94	0.12	Oct-95	-0.98	Minimum	-1.03
May-92	-0.53	Jul-93	-0.71	Sep-94	0.11	Nov-95	-0.76		
Jun-92	-0.62	Aug-93	-0.48	Oct-94	-0.04	Dec-95	-0.85	Maximum	0.31
Jul-92	-0.69	Sep-93	-0.48	Nov-94	-0.09	Jan-96	-0.65		
Aug-92	-0.75	Oct-93	-0.75	Dec-94	-0.07	Feb-96	-0.37		

The results in table 10 show that a portfolio made of stocks with a low free float in a given month has the higher rate of return in the subsequent year if the liquidity premium is high (in absolute value) in the month of formation (Panel B). The reverse is observed when the liquidity premium is low: the rate of return of a high free float portfolio is the higher one in the following year.

Table 10
Liquidity premium, free float, and subsequent returns

Panel A : 19 months with liquidity premium >-0.27

	Free float			All	Nb	T test
	Low	2	High	stocks	obs	(low-high)
1 month	-0,69%	-0,44%	-0,49%	-0,54%	2 016	-0,47
3 months	-3,65%	-3,42%	-2,99%	-3,35%	2 014	-1,16
6 months	-6,18%	-3,28%	-3,63%	-4,36%	2 013	-3,09
9 months	-5,91%	-1,03%	-2,35%	-3,09%	2 011	-3,31
12 months	-7,10%	0,30%	-2,33%	-3,04%	2 005	-3,61

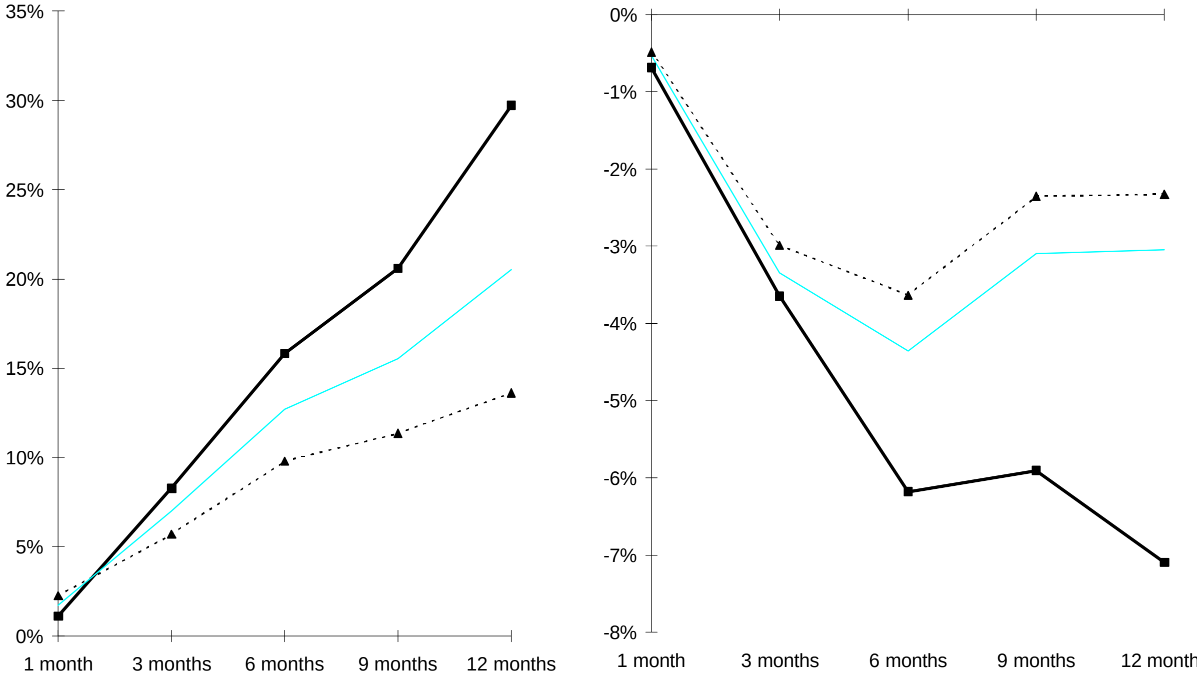
Panel B : 24 months with liquidity premium <-0.59

	Low	2	High			
1 month	1,12%	1,73%	2,25%	1,70%	2 082	-2,0
3 months	8,27%	7,03%	5,71%	6,99%	1 959	2,9
6 months	15,82%	12,60%	9,80%	12,71%	1 953	5,0
9 months	20,59%	14,80%	11,35%	15,54%	1 577	5,6
12 months	29,72%	18,43%	13,62%	20,52%	1 453	7,5

Note : portfolios are constituted from July 1991 to August 1996, following the value of the expected liquidity premium. Ex-post returns are computed from August 1991 to December 1996.

Figure 14
Liquidity premium, free float, and subsequent return

Months with liquidity premium < -0.59 Months with liquidity premium > -0.27



Note : the solid line represents the evolution of the rate of return in the year following the formation month for stocks with a low free float. The dotted line represents the evolution for stocks with a high free float.

When the liquidity premium is high, the market prices the liquidity, and the holding of illiquid stocks (low free float) is rewarded.

6. Conclusions and Perspectives

In this paper, we have discussed the various arguments that support the existence of a liquidity premium whereby the required rate of return on less liquid stocks should be higher than on more liquid stocks. All possible measures of liquidity were reviewed. None are totally satisfactory and we selected relative spread, capitalization, and free float as a proxy for liquidity.

The empirical investigation has used both expectational data for the CAPM measures of risk and return on a universe of more than hundred French stocks between 1991 and 1996 and time-stamped data from the Paris stock market (Société des Bourses Françaises) for the exogenous measures of liquidity.

Indeed, expected returns decrease with free float and capitalization and increase with relative spread. Holding risk constant, these relationships remain. This is an indication that the market beyond risk prices liquidity.

The rates of return observed after the formation month confirm the existence of a small size effect on the Paris Bourse. Portfolios with the lowest free float are more profitable in the following year.

Subsequent returns are directly associated with the value of the liquidity premium in a given month. When the liquidity premium is high (in absolute value), the subsequent returns are highest for portfolios with a low free float, while the reverse is true in months where the liquidity premium is positive or near zero.

These last results corroborate our measurements of liquidity and liquidity premium.

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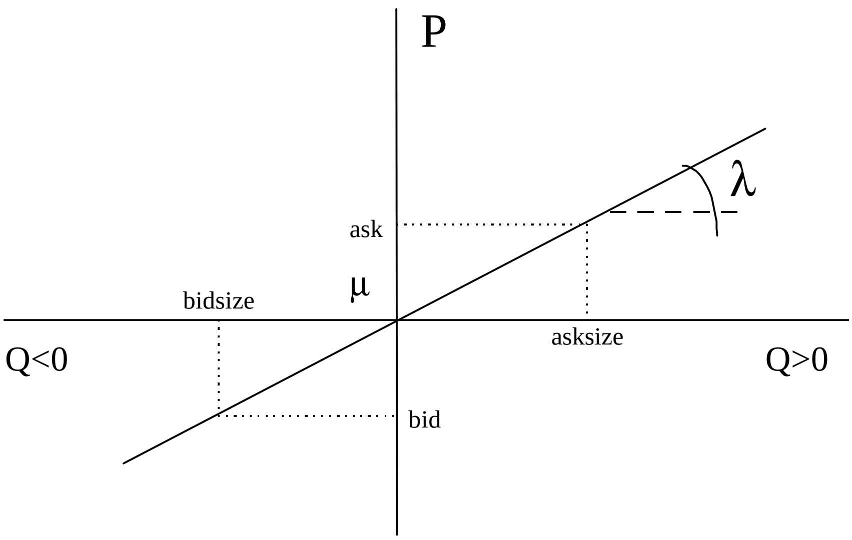
Appendix

Capitalization	At a given point in a session, capitalization is equal to the product of the observed price and the number of shares on the exchange.
Free float	Estimated monthly by Associés en Finance, the free float represents the capitalization that is not frozen in portfolios as a strategic participation. It is the capitalization effectively available for exchange, estimated in millions of French francs.
Lambda	A hypothesis of linearity between the price (P) and transaction volumes (Q) allows:

$$P = m + l \times Q$$



Q has a sign: it is positive if the transaction is initiated by the buyer and it is negative if the transaction is initiated by the seller. μ represents the informational value of the stock. For a very liquid stock (λ=0), there is confusion at every instant between the informational value and the price. For a less liquid stock (λ>0), a transaction has an impact on the price, even if the informational value remains the same: the price goes down if the transaction is initiated by a seller and the price goes up if it is initiated by a buyer. Handa (1993) and Hamet (1995) suggest an estimate of the parameters of the line by supposing that the best limits define the coordinates of two points.

$$l = \frac{ask - bid}{asksize + bidsize}$$

MEC	The market efficiency coefficient (Hasbrouck and Schwartz, 1988) is estimated here as the ratio of the variance of the daily rate of return, estimated from open to close, to seven times the variance of hourly rates of return: $$MEC = \frac{s_{o-c}^{2}}{7 \times s_{h}^{2}}$$
Rate of Return (TRE)	The rate of return required by the market is given by a forecasting market line. The equation of the line is determined monthly from a cloud of forecasts of risk and individual rates of return. Once the forecast risk is established, the forecast rate of return in the CAPM sense is obtained at a glance for a given stock and a given month (see figure 1).
Rate of Return (TRI)	A first forecast rate of return is estimated by Associés en Finance by equalizing the observed price and the sequence of future dividends anticipated by financial analysts. This sequence is estimated from a rate of distribution, a forecast of the EPS for the next fiscal year, the EPS growth rate for the next five years, and a model beyond those five years.
Rate of Return (ex-post)	The rate of return is estimated on hourly data, without excluding the intersession. The rate of return of the intersession is corrected to take into account ex-dividends and the adjustment coefficients due to corporate activity.
Risk	The forecast risk is established for each stock on a monthly basis by Associés en Finance for its Security Market Line service.
Size	The size of a particular transaction is the ratio of the number of shares exchanged in the transaction to the number of transactions reported. An average is then computed by session and by stock.
Spread	The spread is observed together with a transaction for a given stock at the beginning of each hour (or as close as possible to the beginning) and at the end of the session (or as close as possible to the end). For each given transaction which is closest to the beginning of a given time period, the posted spread simultaneous or just prior to the price is identified. The best superior and inferior limits of the order book before the transaction are called *ask* and *bid*. The price is the price at the transaction. The spread is calculated in the following manner: $$\text{Spread} = \frac{2 \times \left| \text{Price} - \frac{\text{ask} + \text{bid}}{2} \right|}{\text{Price}}$$
Transactions	Transactions in French francs during a given hour are estimated by adding the products of the transaction prices and the number of shares exchanged at those prices. An average is then calculated by session and by stock.
Volatility	Standard deviation of the hourly rate of return.
Volume	The volume for a given hour is calculated as the sum of the shares exchanged during the period.

Contents